X-507-66-247

NASA TM X-55571

ANALYSIS OF THE CALIBRATION TESTS FOR THE APOLLO VARIAN RUBIDIUM FREQUENCY STANDARDS

BY

C. W. MURRAY, JR.

MAY 1966

NASA

GODDARD SPACE FLIGHT CENTER
GREENBELT, MD.

X-507-66-247

ANALYSIS OF THE CALIBRATION TESTS FOR THE APOLLO VARIAN RUBIDIUM FREQUENCY STANDARDS

By
C. W. Murray, Jr.

May 1966

Goddard Space Flight Center
Greenbelt, Maryland

ANALYSIS OF THE CALIBRATION TESTS FOR THE APOLLO VARIAN RUBIDIUM FREQUENCY STANDARDS

by

Charles W. Murray, Jr.

SUMMARY

Results of calibration tests conducted by the Bureau of Standards on three R-20 Rubidium Frequency Standards submitted by Varian Associates of Palo Alto, California are analyzed. The technique of least squares is used to calculate "best fit" zero, first, second, and third degree polynomials which describe as a function of time the variation in relative frequency of each standard from a reference – the United States Frequency Standard. A pooled sample of deviations of the daily readings from the daily averages is compared to a normal distribution having the same mean and standard deviation. This is done for each unit. In addition, the residuals of the "best fit" straight lines for each frequency standard are compared to a normal distribution. The deviations of the daily readings from the daily averages as well as the residuals of the "best fit" straight lines show fairly good agreement with a normal distribution.

Using the expressions for the "best fit" straight lines, the drifts in relative frequency over a period of one year are: $-(13.6)10^{-11}$, $+(34.1)10^{-11}$, and $+(1.7)10^{-11}$.

The standard deviations of the polynomials are determined for each frequency unit and plotted as a function of time. It is shown that the straight line is a more "stable predictor" of Rubidium Frequency Standard performance than a higher degree polynomial in the sense that the standard deviation of the straight line remains within an arbitrary tolerance of 1 part in 10^{11} for a much longer period of time (well over a year) than does a higher degree polynomial.

The 90% and 95% tolerance limits for the deviations of the daily readings from the "best fit" straight lines are also shown for each frequency standard.

CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

ANALYSIS OF THE CALIBRATION TESTS FOR THE APOLLO VARIAN RUBIDIUM FREQUENCY STANDARDS

INTRODUCTION

Results of calibration tests, performed by the National Bureau of Standards, on three R-20 Rubidium Frequency Standards (serial 100, 106, and 107) submitted by Varian Associates of Palo Alto, California, have been analyzed. Data on serial 100 covered the period from 22 July 1965 to 15 November 1965; data on serial 106 covered the period from 1 September 1965 to 6 December 1965; and, data on serial 107 covered the period from 23 August 1965 to 15 November 1965.

An explanation of the test procedure is given below as well as a discussion of the least squares polynomial fitting to the data, a statistical analysis of the deviations of the daily readings from the daily averages, and the determination of the standard deviation of the polynomial. A section is devoted to a discussion of the least squares technique and the determination of the covariance matrix of the coefficients.

The most salient feature of the analysis is that a straight line fit to the data provides us with a more "stable predictor" of frequency standard performance than does a higher degree polynomial fit.

TEST PROCEDURE

A simplified block diagram of the Rubidium Frequency Standard Calibration Test is shown in Figure 1. Letting t = time in seconds, A_0 = amplitude of the signal from the R-20 (volts), and $\phi_0(t)$ = phase angle of the R-20 (radians), we can write an expression for the signal from the R-20 oscillator, $V_0(t)$, as:

$$V_0(t) = A_0 \cos(\phi_0(t)) \tag{1}$$

where

$\phi_0(t) = 2\pi f_0 t + \varphi_0(t)$ (radians)

f_0 = R-20 nominal frequency = 5MHz

$\varphi_0(t)$ = variation of R-20 phase angle due to random noise, frequency offset, and drift (radians)

The signal from the R-20 Test Oscillator is fed into a frequency multiplier (K) and the output signal, $V_1(t)$, is:

$$V_1(t) = A_0 \cos [K \phi_0(t)]. \tag{2}$$

$V_1(t)$ is then fed into a phase detector. A signal, $V_2(t)$, from the USFS (United States Frequency Standard – Cesium) with a nominal frequency f_s = 9192.631770 MHz (Reference 1), amplitude A_2 (volts), and phase angle $\phi_s(t)$ (radians), is also fed into the phase detector. $V_2(t)$ can be represented by:

$$V_2(t) = A_2 \cos [\phi_s(t)] \tag{3}$$

where

$\phi_s(t) = 2\pi f_s t + \varphi_s(t)$ (radians)

$\varphi_s(t)$ = variation in phase angle of the USFS due to random noise (radians)

The output difference signal from the phase detector, $V(t)$, then becomes:

$$V(t) = A \cos [2\pi(f_s - Kf_0)t + (\varphi_s(t) - K\varphi_0(t))] \tag{4}$$

where

A = amplitude (volts)

K = multiplying factor

Letting $\varphi_0(0)$ and $\varphi_s(0)$ represent respectively the values of $\varphi_0(t)$ and $\varphi_s(t)$ at time $t = 0$, the change in phase angle, $\Delta\varphi$, at the output of the detector, after a time $t = T$ seconds may be written:

$$\Delta\varphi = \varphi_s(T) - K\varphi_0(T) - [\varphi_s(0) - K\varphi_0(0)] \tag{5}$$

or

$$\Delta\varphi \doteq K\Delta\varphi_0$$

where

$$\varphi_s(T) - \varphi_s(0) \text{ is small}$$

and

$$\Delta\varphi_0 = \varphi_0(0) - \varphi_0(T)$$

The change in phase angle, $\Delta\varphi_0$, due to random noise, frequency offset, and frequency drift in the R-20 over an interval of $T = 200$ seconds (a time interval corresponding to 10 cycles of the difference frequency), was measured and divided by T to obtain:

$$\Delta f_0 = \frac{\Delta\varphi_0}{T} \doteq \left(\frac{1}{K}\right) \frac{\Delta\varphi}{T}$$

or

$$\frac{\Delta f_0}{f_0} \doteq \left(\frac{1}{K}\right) \frac{\Delta\varphi}{f_0 T} \tag{6}$$

Each day approximately 6 to 12 observations as given by equation (6) were made.

The values as recorded in references 2, 3, and 4 are expressed in $(\Delta f_0/f_0)$ 10^{10} units. In the same references, it is stated that the absolute accuracy of the USFC is presently believed to be $\pm 6 \times 10^{-12}$. No indication of the meaning of this number is given.

ANALYSIS AND DISCUSSION

In the following paragraphs we will discuss the analysis techniques and the results obtained in this report.

Least Squares Polynomial Fitting to Daily Averages

The daily averages[1] were calculated using all of the daily observations except in those instances where an observation varied significantly from the others. In these cases the particular observation was thrown out and the average computed using the remaining observations. Figures 2, 3, 4, and 5 show those observations which were excluded. It should be noted that these observations were not thrown out by any fixed criterion but rather by an "eye-smoothing" process.

A least squares polynomial was then fitted to the observed daily averages for each Rubidium Standard. These results may be seen in Figures 6 through 11 inclusive. Figures 6, 8, and 10 which refer to serials 100, 106, and 107 respectively contain plots of the daily averages versus time (in days) as well as the "best fit" (in the sense of least squares) polynomials of degree 0, 1, 2, and 3 for these data points. Figures 7, 9, and 11 also pertain to serials 100, 106, and 107 respectively, but are for extended periods of time – up to one year. In each figure the equation of the "best fit" polynomial is indicated as well as the standard

[1]The daily averages were used instead of the daily readings since there were no recordings of the time of the daily readings.

deviation of fit, η.[2] It should be noted that two daily averages were excluded in order to "smooth out" the data (see Figures 8 and 10).

Using the expressions for the "best fit" straight lines, the drifts in relative frequency over a period of one year (T = 365) are: $-(13.6)10^{-11}$ for serial 100; $+(34.1)10^{-11}$ for serial 106; and, $+(1.7)10^{-11}$ for serial 107.

Statistical Analysis of Daily Variation and Variation of the Residuals of the "Best Fit" Straight Line

A pooled sample of deviations of the daily readings from the daily averages was taken for each frequency standard (106 for serial 100, 110 for serial 106, and 107 for serial 107) and the standard deviation calculated. Figure 12 shows the histograms for the samples compared to normal distributions having the same mean (zero) and standard deviation.

Figures 13, 14, and 15 show the histograms of the residuals or the deviations of the daily averages from the "best fit" straight lines. Again the histograms are compared with normal distributions having the same mean and standard deviation. For a normal distribution the expected number of observations falling outside ±1.64 standard deviations (these limits correspond to a probability of 90%) is 4.5 for a sample size of 45 (serial 100), 3.8 for a sample size of 38 (serial 106), and 2.9 for a sample size of 29 (serial 107). The observed numbers which fell outside these limits were 5, 3, and 2 for serial 100, 106, and 107, respectively. It does not seem unreasonable, therefore, to assume a normal distribution for the residuals.

Figures 16, 17, and 18 show the 90% and 95% tolerance intervals for the deviations of the daily readings from the "best fit" straight lines for the three units. In calculating these limits a normal distribution was assumed for the residuals. The standard deviation of the daily readings from the straight line was calculated using the following relationship:

$$s = \sqrt{\eta^2 + s_d^2} \tag{7}$$

[2] The standard deviation of fit η is computed as:

$$\eta = \sqrt{\left(\frac{1}{n-\mu}\right) \sum_{i=1}^{n} [y_i - P(x_i)]^2}$$ where n = number of data points,

$\mu = k + 1$, k = degree of the polynomial, $P(x_i)$ = value of the "best fit" polynomial at the point x_i ($i = 1, 2, \ldots n$), y_i = observed value corresponding to x_i (see Reference 5).

where η = standard deviation of fit and s_d is the standard deviation of the pooled sample.[3]

The Standard Deviation of the Least Squares Polynomial As a Function of Time[4]

Having calculated the coefficients of a least squares polynomial to a set of experimental data, one may ask, "How good is the polynomial in predicting the performance of the Rubidium Standards?" Or, phrasing the question another way, "What is the spread of values that the polynomial can take as a function of time based upon the finite sample of data points?" In order to answer this question, it will be sufficient to write an expression for the standard deviation of the polynomial at a given instant of time.

Let the form of the least squares polynomial be

$$\bar{y} = P(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_k x^k \qquad (k \geq 0) \tag{8}$$

When $x = x_j$, $\bar{y}$ will have the value $\bar{y}_j$ given by

$$\bar{y}_j = P(x_j) = a_0 + a_1 x_j + \cdots + a_k x_j^k \tag{9}$$

Considering the coefficients as random variables we can write an expression for the standard deviation of $\bar{y}$ when $x = x_j$ as follows:

$$\sigma_{\bar{y}_j} = \sqrt{2\sum_{m=0}^{k}\sum_{n=0}^{k-m} x_j^{2m+n}\,\sigma_{a_m a_{m+n}} - \sum_{m=0}^{k} x_j^{2m}\,\sigma_{a_m}^2} \tag{10}$$

where $\sigma^2_{a_m} = \sigma_{a_m a_m}$ is the variance of a_m and $\sigma_{a_m a_{m+n}}$ is the covariance between a_m and a_{m+n}.

---

[3] This relationship holds if it is assumed that the statistics do not change with time and that the deviations of the daily readings from the "best fit" straight line are uncorrelated.

[4] A rather detailed description of the known "Method of Least Squares" is presented for the convenience of the reader.

As an example, the standard deviation of $\bar{y}_j$ for a second degree polynomial when $x = x_j$ can be written as

$$\sigma_{\bar{y}_j} = \sqrt{\sigma^2_{a_0} + x_j^2 \sigma^2_{a_1} + x_j^4 \sigma^2_{a_2} + 2x_j \sigma_{a_0 a_1} + 2x_j^2 \sigma_{a_0 a_2} + 2x_j^3 \sigma_{a_1 a_2}} \tag{11}$$

In order to determine the variances and covariances in (10) we will briefly describe the least squares technique using matrix notation in order to simplify the algebra.

Brief Discussion of the Least Squares Technique and the Determination of the Covariance Matrix of the Coefficients

Reference (5) contains a discussion of the generalized case of the method of least squares and, in addition, includes a procedure for calculating the uncertainty associated with each quantity determined by the least squares method. In this report the quantities are the coefficients of the "best fit" polynomials and the uncertainty is the standard deviation of each coefficient. We will now briefly describe the least squares method and determine the covariance matrix of the coefficients from which the uncertainties associated with the coefficients as well as the covariances between the coefficients can be obtained.

Let the data be represented by the set of n points (x_1, y_1), (x_2, y_2), . . . , (x_n, y_n) where it is assumed that the errors are in the ordinates y_i only and let the form of the polynomial be as indicated in (8) with $n \geq k$. The least squares method assumes that the "best" set of values for the coefficients a_i is that set which minimizes the sum S of the squares of the deviations (v_i's) of the observed quantities (y_i's) from the values of the polynomial ($\bar{y}_i$'s). This can be expressed mathematically as

$$S = \sum_{i=1}^{n} v_i^2 = \sum_{i=1}^{n} (y_i - \bar{y}_i)^2 \tag{12}$$

In order to minimize S it is necessary to take the partial derivatives of S with respect to each of the coefficients and set these derivatives equal to zero. When this is done $k+1$ "normal" equations in $k+1$ unknowns result. The set of coefficients is then the set of roots which satisfy the $k+1$ simultaneous linear equations. The normal equations are given by:

$$(n)\quad a_0 + \left(\sum_{i=1}^{n} x_i\right) a_1 + \cdots + \left(\sum_{i=1}^{n} x_i^k\right) a_k = \sum_{i=1}^{n} y_i$$

$$\left(\sum_{i=1}^{n} x_i\right) a_0 + \left(\sum_{i=1}^{n} x_i^2\right) a_1 + \cdots + \left(\sum_{i=1}^{n} x_i^{k+1}\right) a_k = \sum_{i=1}^{n} y_i\, x_i \tag{13}$$

$$\vdots$$

$$\left(\sum_{i=1}^{n} x_i^k\right) a_0 + \left(\sum_{i=1}^{n} x_i^{k+1}\right) a_1 + \cdots + \left(\sum_{i=1}^{n} x_i^{2k}\right) a_k = \sum_{i=1}^{n} y_i\, x_i^k$$

The set of equations in (13) can be solved by standard techniques. For ease of notation let us now use matrices. First, we can write down n "observational" equations which express each observation as a function of the least squares polynomial evaluated at the point x_j and the residual v_j $(j = 1, 2, \ldots n)$.

$$\begin{aligned} y_1 &= a_0 + a_1 x_1 + \cdots + a_k x_1^k + v_1 \\ &\vdots \\ y_n &= a_0 + a_1 x_n + \cdots + a_k x_n^k + v_n \, . \end{aligned} \tag{14}$$

Letting[5]

$$\mathbf{Y} = \begin{bmatrix} y_1 \\ y_2 \\ \\ \vdots \\ \\ y_n \end{bmatrix} \quad \mathbf{A} = \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^k \\ 1 & x_2 & x_2^2 & \cdots & x_2^k \\ & & & & \\ \vdots & & & & \\ & & & & \\ 1 & x_n & x_n^2 & \cdots & x_n^k \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} a_0 \\ a_1 \\ \\ \vdots \\ \\ a_k \end{bmatrix}$$

[5] $\mathbf{A}^T$ denotes the transpose of $\mathbf{A}$; $\mathbf{A}^{-1}$ denotes the inverse of $\mathbf{A}$; $\mathcal{E}(\mathbf{V})$ denotes the expectation of a random matrix $\mathbf{V}$. $\mathcal{E}(\mathbf{V}\mathbf{V}^T)$ denotes the covariance matrix of a random matrix $\mathbf{V}$ with zero mean.

and

$$\mathbf{V} = \begin{bmatrix} v_1 \\ v_2 \\ \cdot \\ \cdot \\ \cdot \\ v_n \end{bmatrix}$$

the equations in (14) become

$$\mathbf{Y} = \mathbf{A}\mathbf{X} + \mathbf{V} \tag{15}$$

The normal equations in (13) may be written as

$$(\mathbf{A}^T \mathbf{A})\ \mathbf{X} = \mathbf{A}^T \mathbf{Y} \tag{16}$$

Solving the matrix equation (16) for $\mathbf{X}$ we obtain the "best" estimate for the coefficients

$$\hat{\mathbf{X}} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{Y} \tag{17}$$

In order to find the covariance matrix of the coefficients we must find the expected value of $(\hat{\mathbf{X}}-\mathbf{X})\ (\hat{\mathbf{X}}-\mathbf{X})^T$. Substituting (15) into (17) one obtains after some manipulation

$$\mathcal{E}(\hat{\mathbf{X}} - \mathbf{X})(\hat{\mathbf{X}} - \mathbf{X})^T = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathcal{E}(\mathbf{V}\mathbf{V}^T)\ \mathbf{A}(\mathbf{A}^T\mathbf{A})^{-1} \tag{18}$$

If we assume the residuals v_i ($i = 1,2, \ldots n$) to be uncorrelated random variables each with variance η^2 (the square of the standard deviation of fit, previously defined) we have

$$\mathcal{E}(\mathbf{V}\mathbf{V}^T) = \eta^2 \mathbf{I} \tag{19}$$

where $\mathbf{I}$ is the $n \times n$ identity matrix.

Substituting (19) into (18) we have the required covariance matrix of the coefficients

$$\mathcal{E}(\hat{\mathbf{X}} - \mathbf{X})(\hat{\mathbf{X}} - \mathbf{X})^T = \eta^2 (\mathbf{A}^T\mathbf{A})^{-1} \tag{20}$$

Letting P denote the covariance matrix of the coefficients, and Q the inverse of $A^T A$, we can write

$$P = \begin{bmatrix} \sigma^2_{a_0} & \sigma_{a_0 a_1} & \cdots & \sigma_{a_0 a_k} \\ \sigma_{a_0 a_1} & \sigma^2_{a_1} & \cdots & \sigma_{a_1 a_k} \\ \sigma_{a_0 a_2} & \sigma_{a_1 a_2} & & \sigma_{a_2 a_k} \\ \vdots & & & \\ \sigma_{a_0 a_k} & \sigma_{a_1 a_k} & & \sigma^2_{a_k} \end{bmatrix} \tag{21}$$

$$= \eta^2 \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1,k+1} \\ q_{21} & q_{22} & \cdots & q_{2,k+1} \\ \vdots & & & \\ q_{k+1,1} & & & q_{k+1,k+1} \end{bmatrix}$$

As an example, the covariance between a_1 and a_2 would be given by $q_{23}\ \eta^2$ and the variance of a_3 would be given by $q_{44}\ \eta^2$.

The variances and covariances of the coefficients were calculated for each degree polynomial (zero, first, second, and third) and for each frequency standard. They were then substituted into (10). The standard deviation of the polynomial was then plotted as a function of x_j and can be seen in Figures 19, 20, and 21.

It is evident from these figures that the uncertainty (standard deviation) of the second and third degree polynomials increases sharply just after the period of observation and that the standard deviation of the straight line stays within an arbitrary tolerance of 1 part in 10^{11} for a much longer period of time (over a year) than the standard deviation of the higher degree polynomial.

Table I gives the standard deviation of the coefficients of the least squares polynomials, and Table II gives the length of time in days during which the standard deviation of the polynomial is less than 1 part in 10^{11}.

CONCLUSIONS

(1) A "best fit" straight line serves as a more "stable predictor" of Rubidium Frequency Standard performance than a higher degree polynomial in the sense that the standard deviation of the straight line remains within an arbitrary tolerance of 1 part in 10^{11} for a longer period of time (over 1 year) than the standard deviation of a higher degree polynomial.

(2) The "best fit" straight lines for Rubidium Frequency Standards R-20 Serials 100, 106, and 107 give the following drifts over a period of one year: $-(13.6)10^{-11}$ (serial 100); $+(34.1)10^{-11}$ (serial 106), and $+(1.7)10^{-11}$ (serial 107).

(3) For periods of time comparable to those over which the observations were made on Rubidium Frequency Standards R-20 Serials 100, 106, and 107, and under the assumptions indicated, there are 90% and 95% probabilities that the daily readings will fall within the boundaries as shown in Figures 16, 17, and 18.

ACKNOWLEDGMENTS

The author wishes to thank Dr. B. Kruger of the Mission Analysis Office for his direction and guidance in the preparation of this report and Dr. Herman Epstein of Bisset-Berman for his suggestion to analyze the standard deviation of the polynomial.

Table I
Standard Deviations* of the Coefficients of the
Least Squares Polynomials

General Form of the Least Squares Polynomial (see Figures 6, 8, and 10).

$$\left(\frac{\Delta f}{f}\right)10^{10} = a_0 + a_1 T + \ldots + a_k T^k$$

where k is the degree of the polynomial and T is time (in days).

	k	σa_0	σa_1	σa_2	σa_3
Serial 100	0	.14600			
	1	.01735	$.2775 \times 10^{-3}$		
	2	.02944	1.228×10^{-3}	$.9842 \times 10^{-5}$	
	3	.04194	3.083×10^{-3}	6.243×10^{-5}	3.541×10^{-7}
Serial 106	0	.29090			
	1	.01703	$.3004 \times 10^{-3}$		
	2	.01408	$.7491 \times 10^{-3}$	$.7929 \times 10^{-5}$	
	3	.01436	1.533×10^{-3}	3.970×10^{-5}	2.817×10^{-7}
Serial 107	0	.04170			
	1	.01299	$.2810 \times 10^{-3}$		
	2	.01543	1.024×10^{-3}	1.292×10^{-5}	
	3	.01914	2.560×10^{-3}	7.780×10^{-5}	6.400×10^{-7}

*All values in the body of the table are in parts in 10^{10}.

Table II
Length of Time in Days During which the Standard Deviation of the Polynomial* is Less Than 1 Part in 10^{11}

	Linear	Quadratic	Cubic
Serial 100	411	165	133
Serial 106	380	161	125
Serial 107	393	129	99

*The standard deviation of a k^{th} degree polynomial when $x = x_i$ is given by the following relationship:

$$\sigma_{\overline{y}_i} = \sqrt{2 \sum_{m=0}^{k} \sum_{n=0}^{k-m} x_i^{2m+n} \sigma_{a_m a_{m+n}} - \sum_{m=0}^{k} x_i^{2m} \sigma_{a_m}^2}$$

where $\sigma_{a_m}^2 = \sigma_{a_m a_m}$ is the variance of a_m and $\sigma_{a_m a_{m+n}}$ is the covariance between a_m and a_{m+n}.

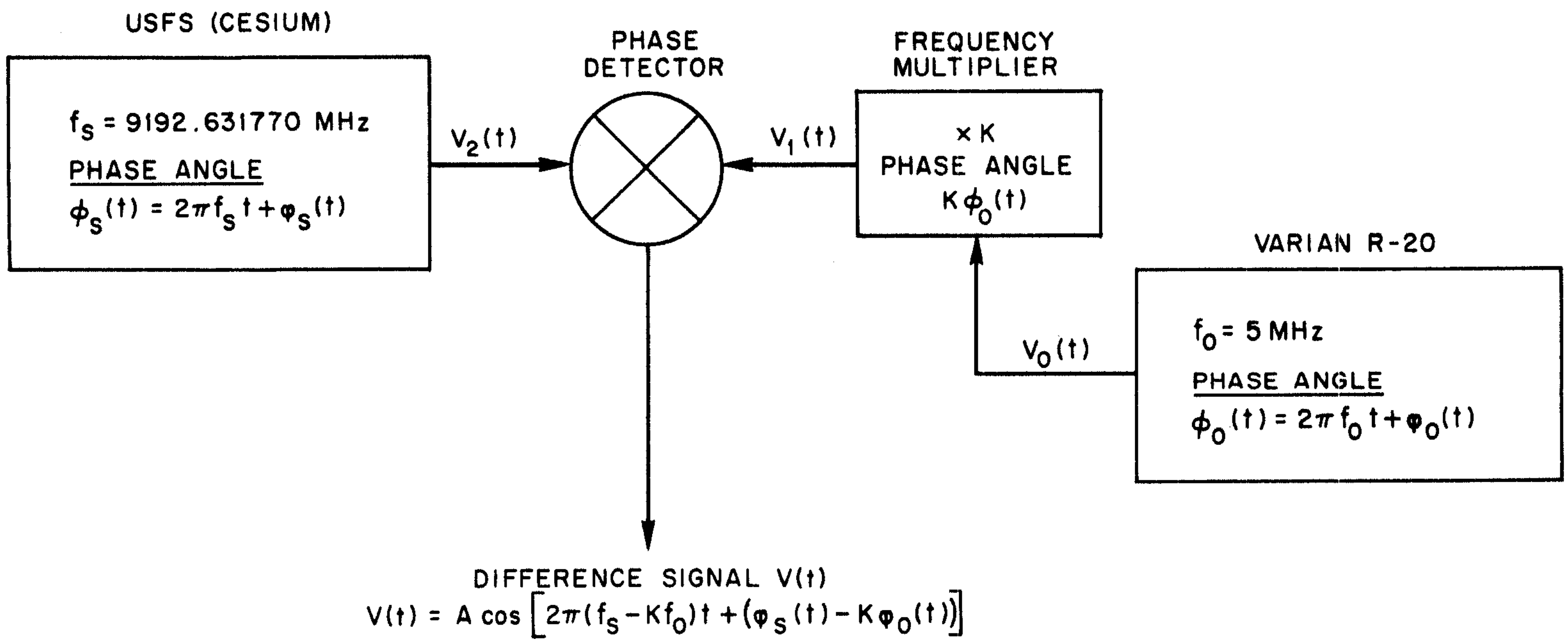


Figure 1–Simplified Block Diagram of Rubidium Frequency Standard Calibration Test

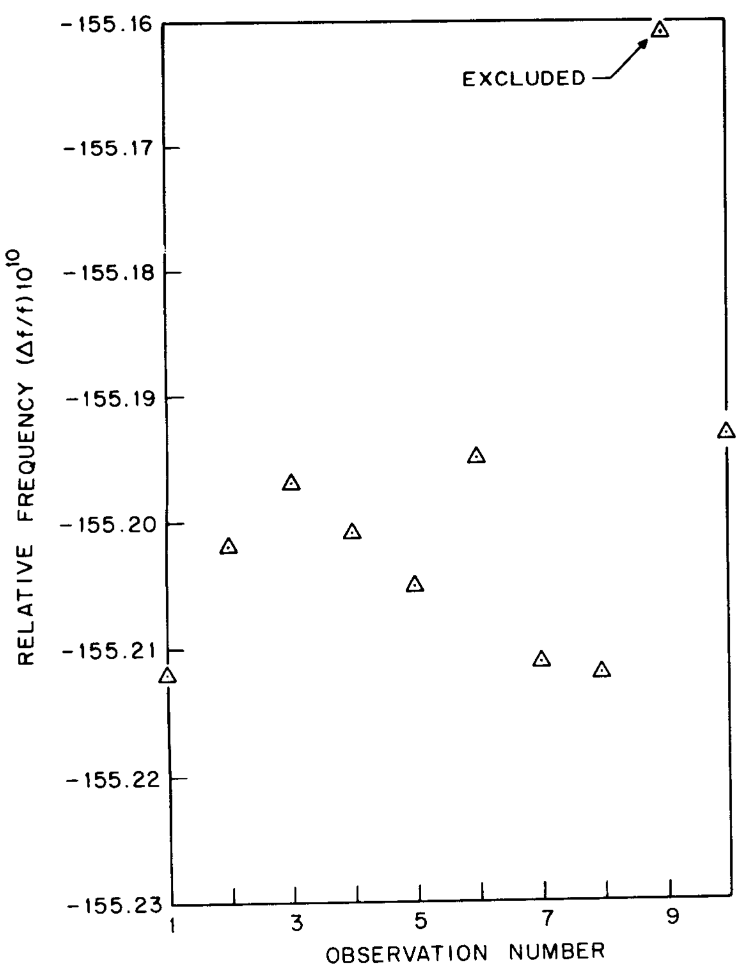


Figure 2–Observations of R-20 Serial 106 Taken on 9/8/65
(200 Second Integration Time for Each Measurement)

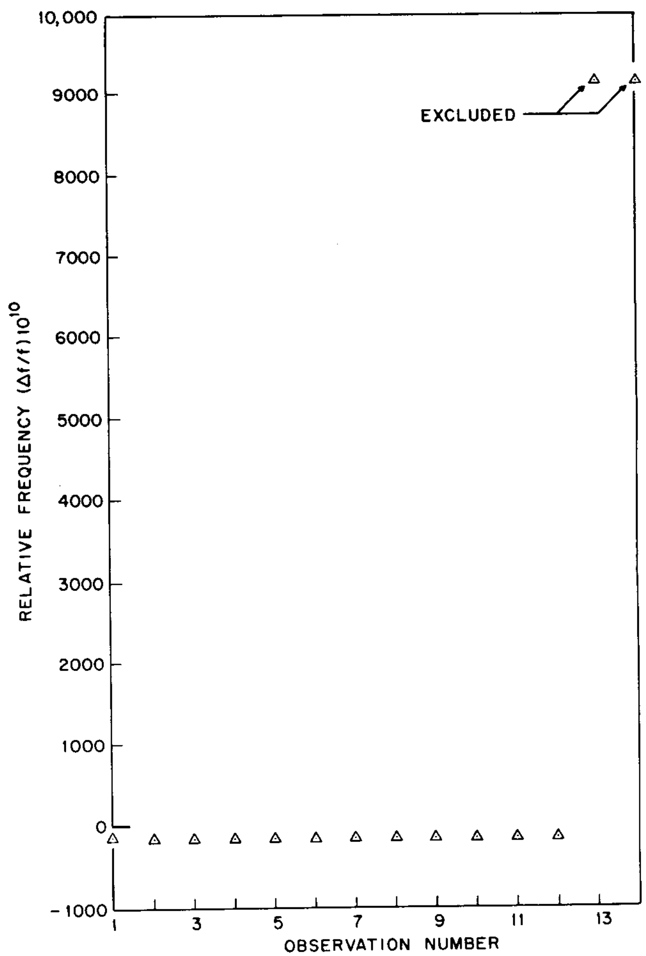


Figure 3–Observations of R-20 Serial 106 Taken on 10/4/65
(200 Second Integration Time for Each Measurement)

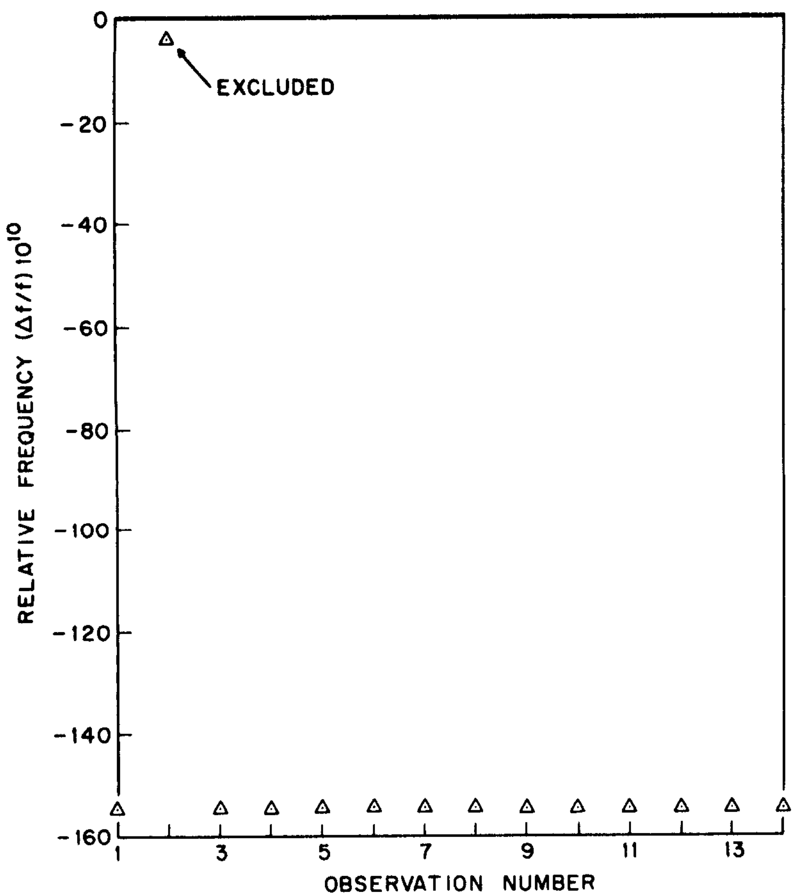


Figure 4–Observations of R-20 Serial 106 Taken on 11/26/65 (200 Second Integration Time for Each Measurement)

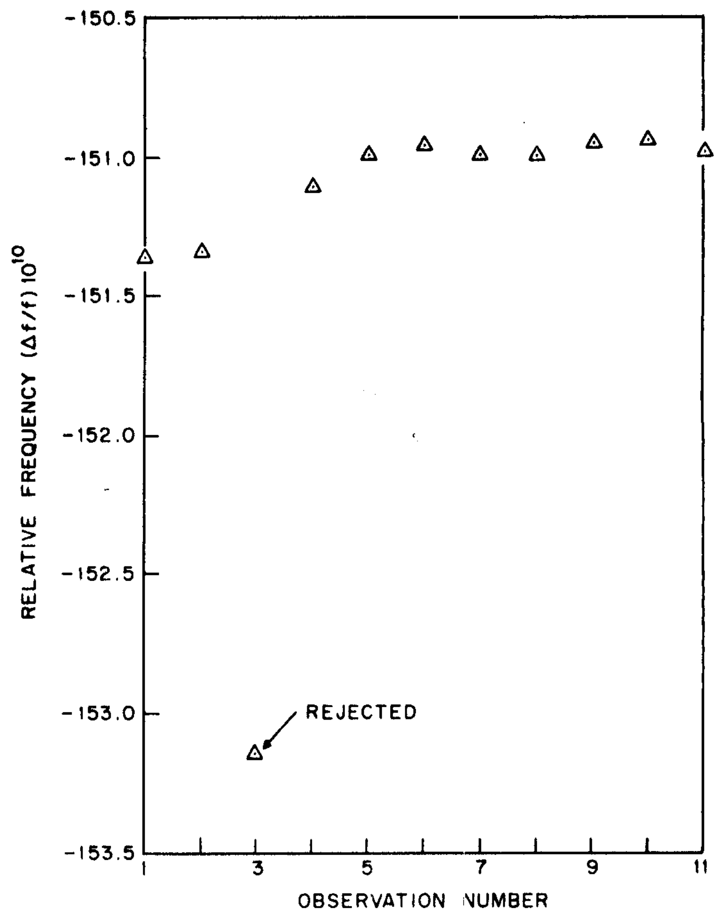


Figure 5–Observations of R-20 Serial 107 Taken on 10/8/65 (200 Second Integration Time for Each Measurement)

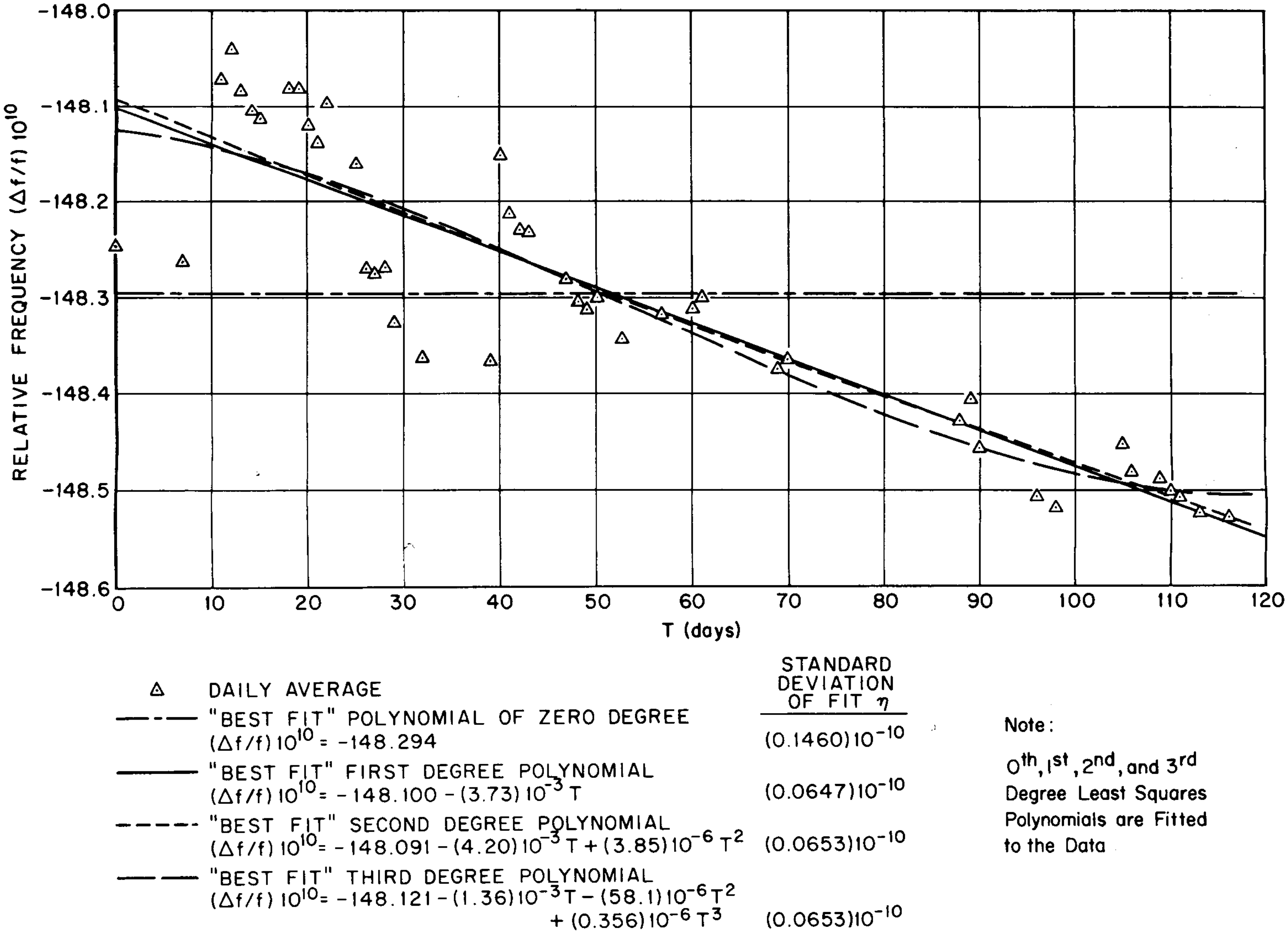


Figure 6–Daily Averages of Relative Frequency Drift of Varian Rubidium Standard R-20 Serial 100

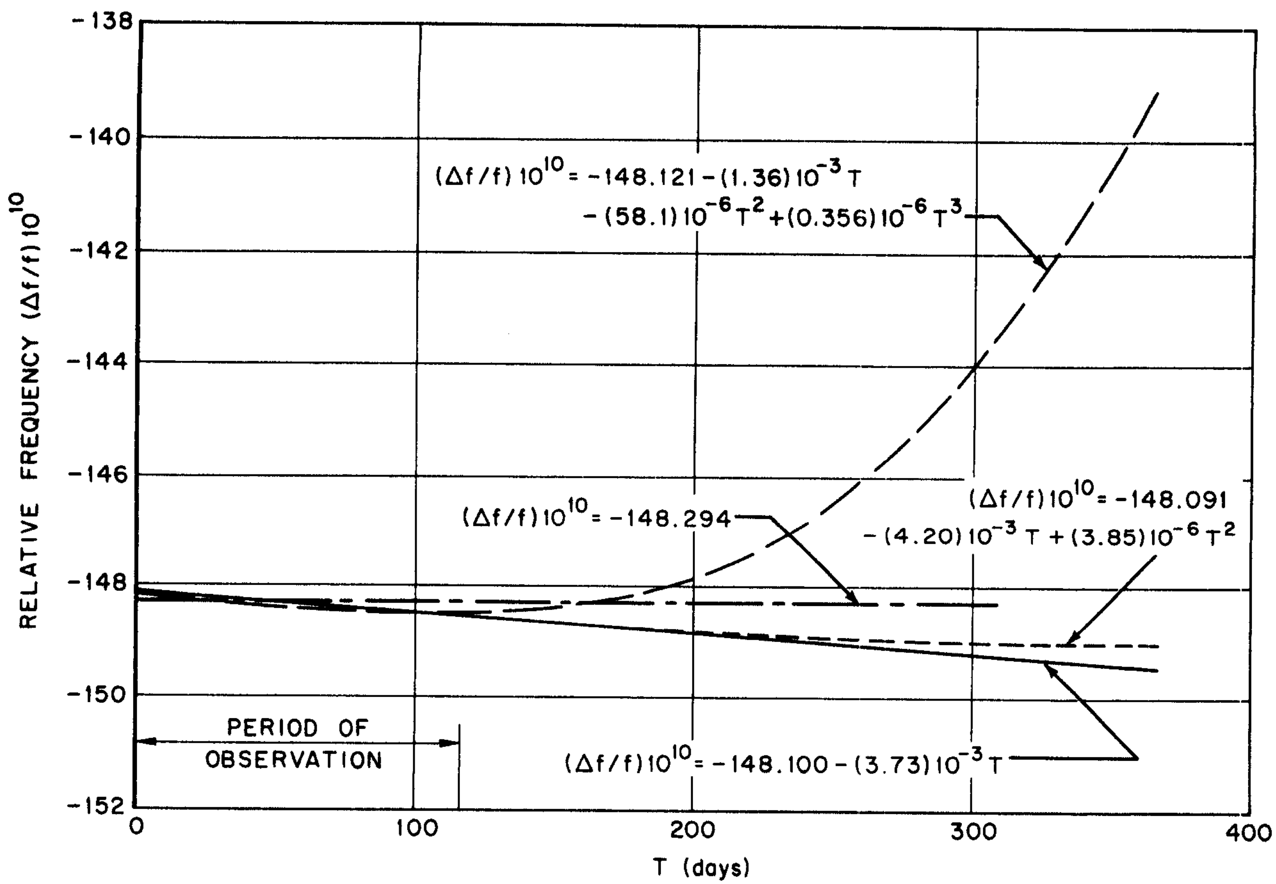


Figure 7–"Best Fit" Least Squares Polynomials of Degree 0, 1, 2, and 3 for Varian Rubidium Standard R-20 Serial 100 (extended time scale)

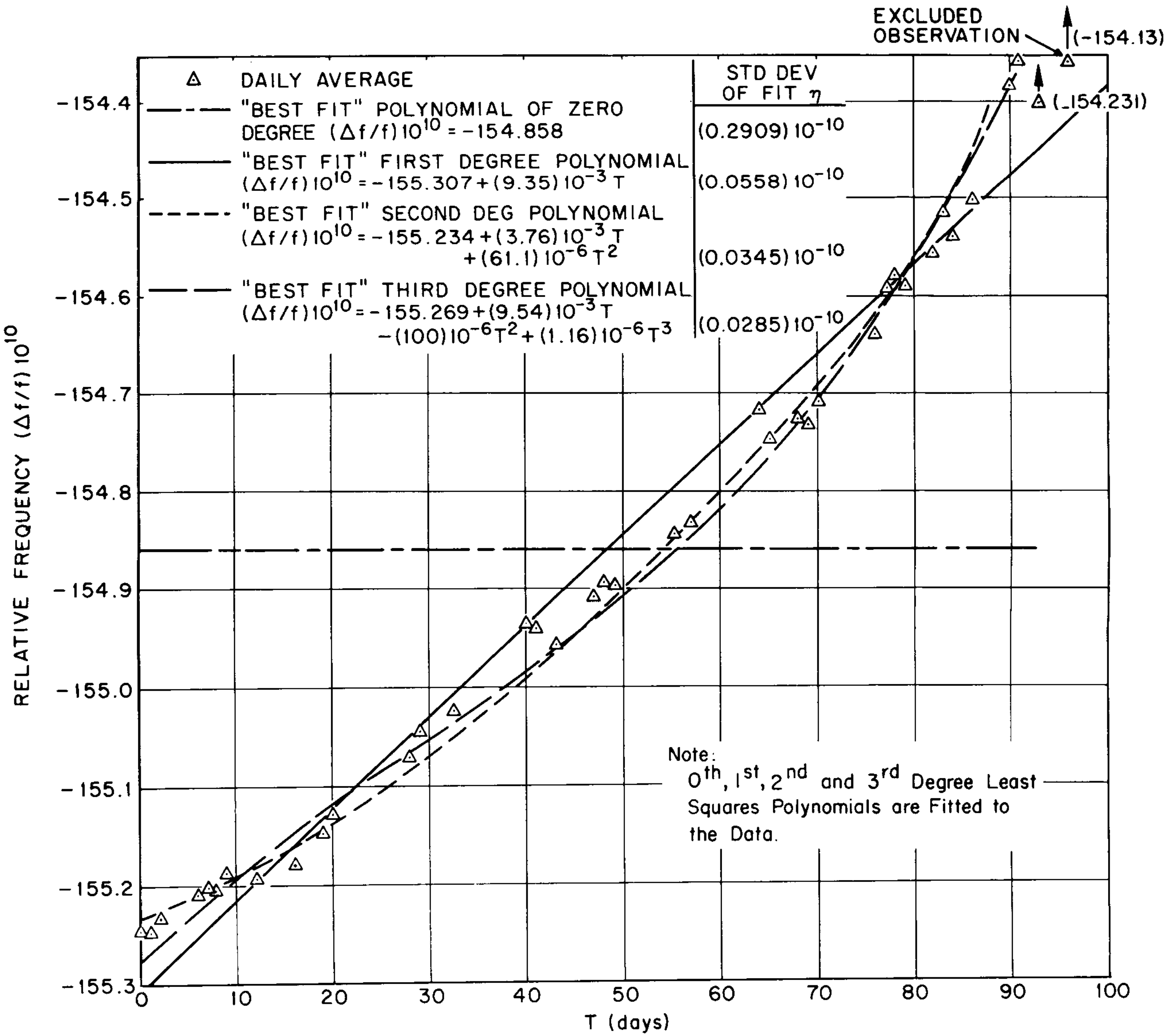


Figure 8–Daily Averages of Relative Frequency Drift of Varian Rubidium Standard R-20 Serial 106

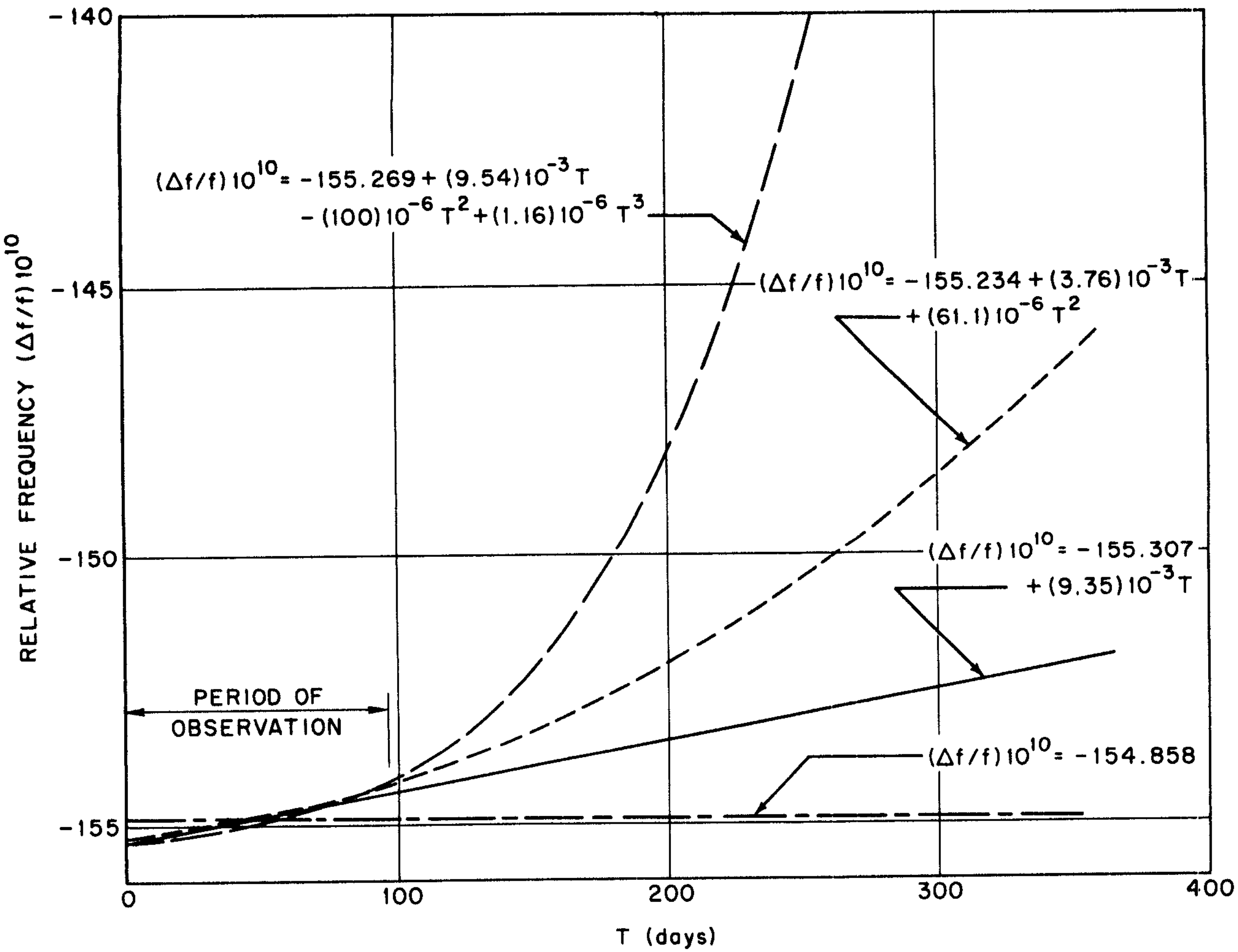
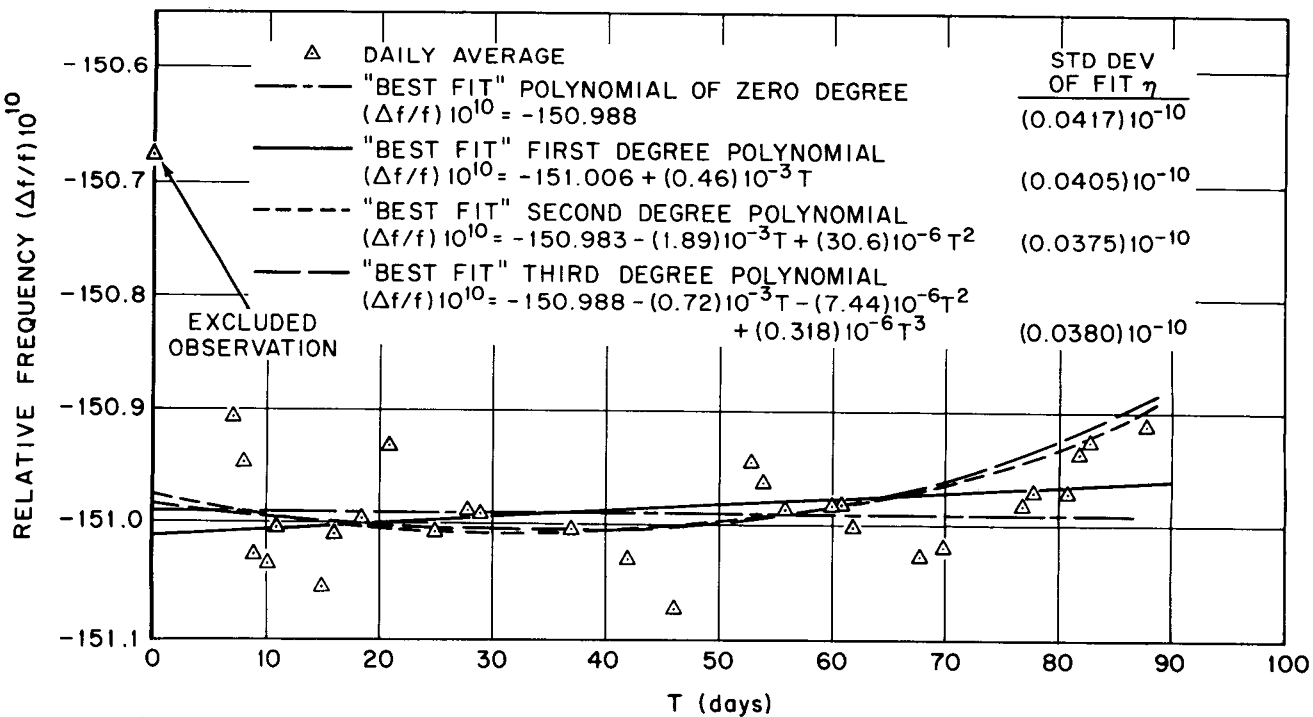


Figure 9–"Best Fit" Least Squares Polynomials of Degree 0, 1, 2, and 3 for Varian Rubidium Standard R-20 Serial 106 (extended time scale)



Note:
0^{th}, 1^{st}, 2^{nd}, and 3^{rd} Degree Least Squares Polynomials are Fitted to the Data.

Figure 10–Daily Averages of Relative Frequency Drift of Varian Rubidium Standard R-20 Serial 107

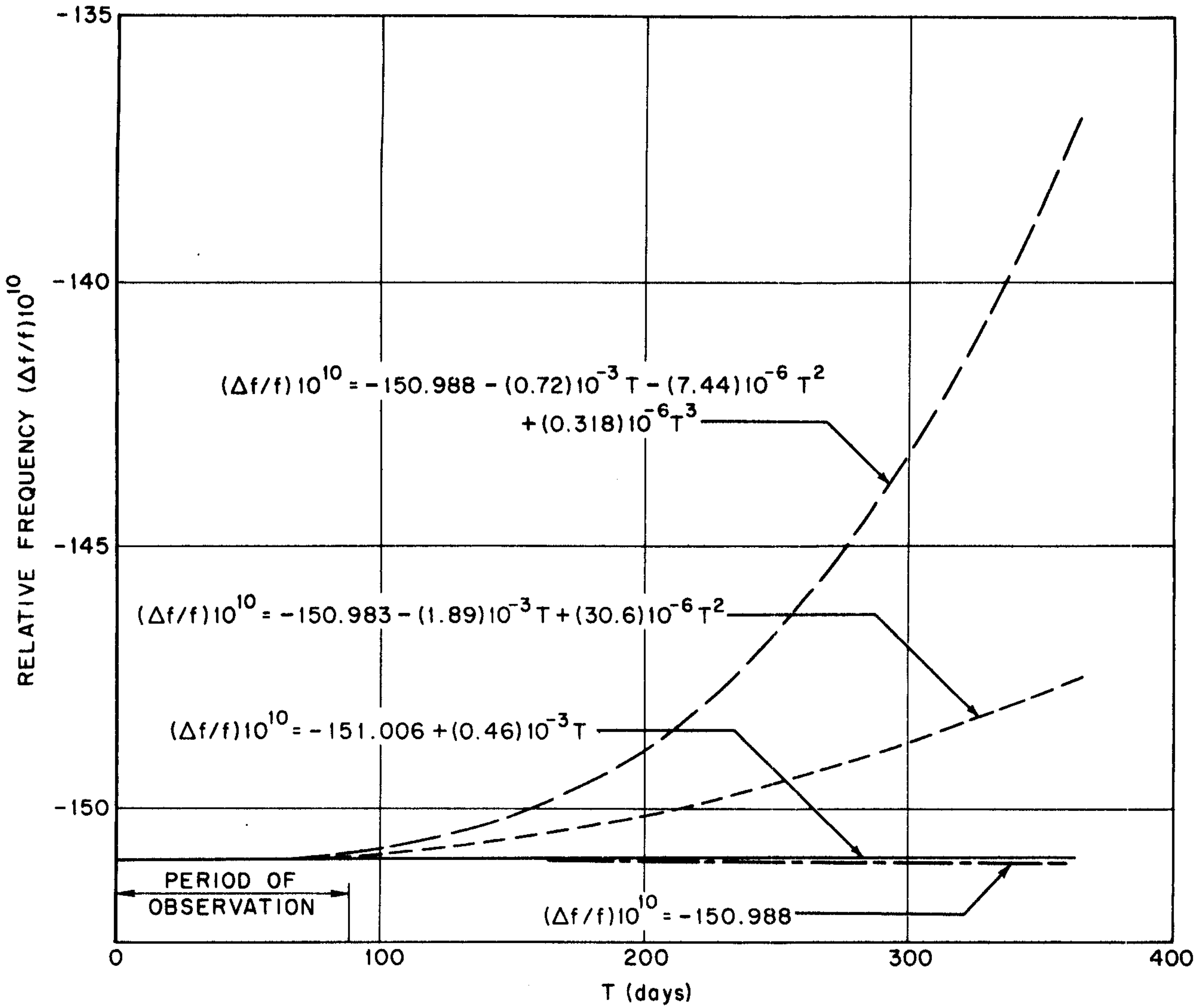
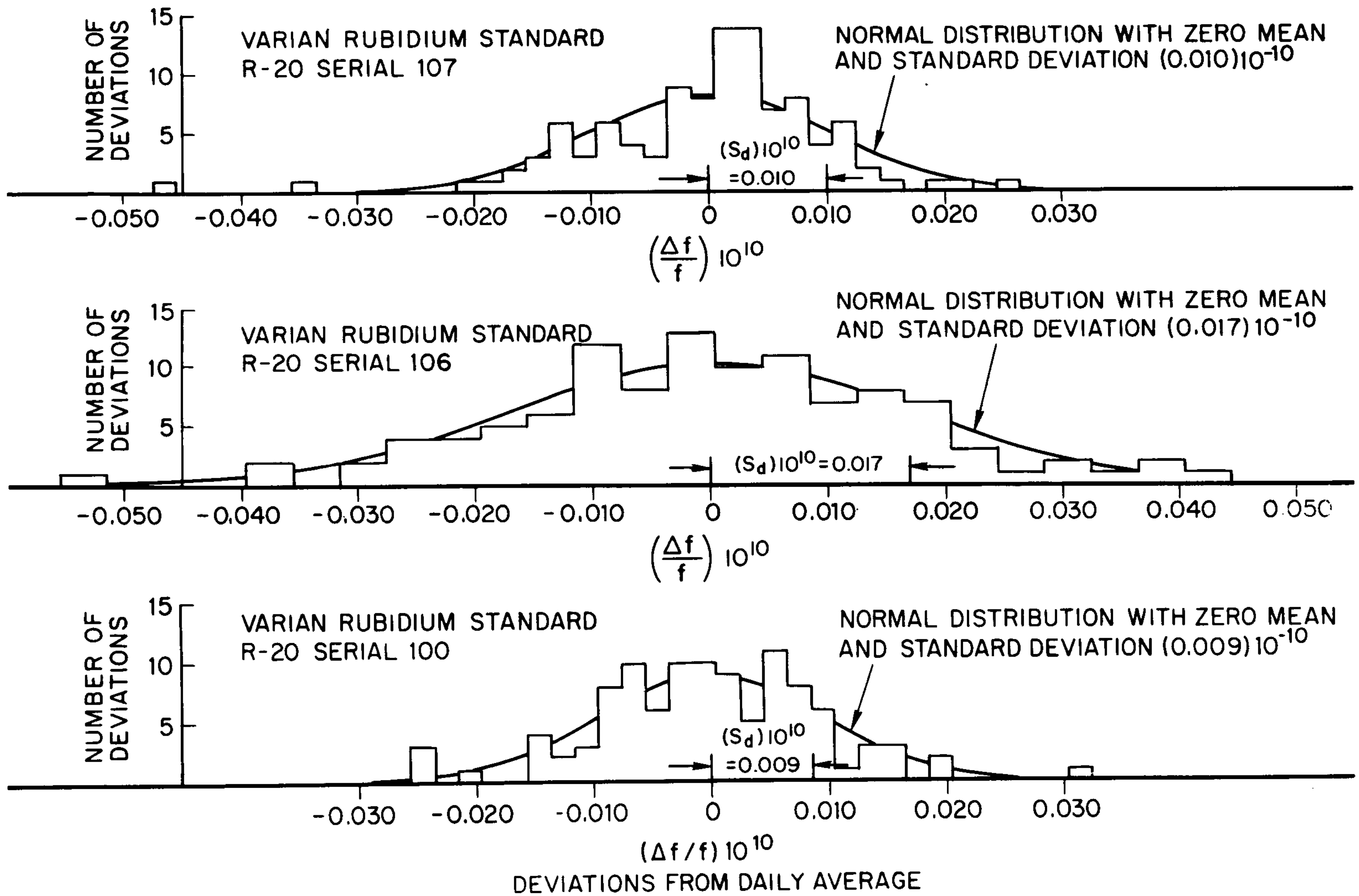


Figure 11—"Best Fit" Least Squares Polynomials of Degree 0, 1, 2, and 3 for Varian Rubidium Standard R-20 Serial 107 (extended time scale)



Note:

Readings for Several Days were Pooled
to Increase the Sample Size

Figure 12—Histograms Showing Spread of Daily Readings

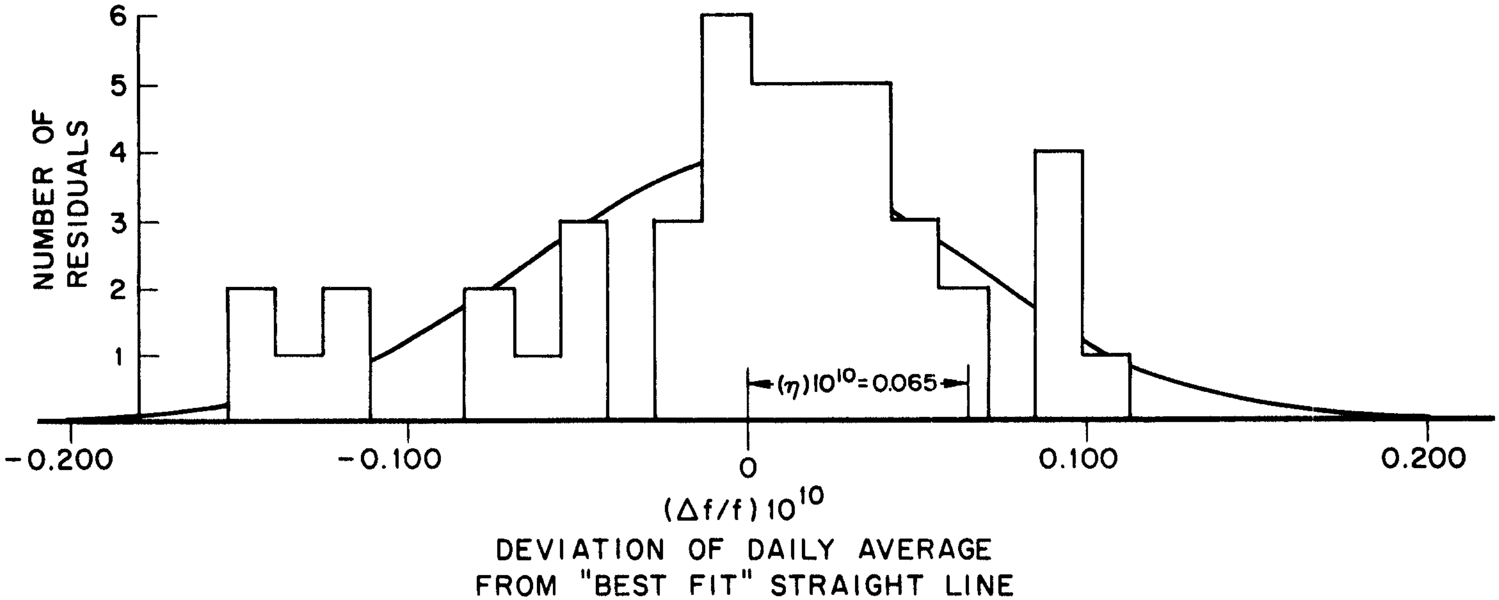


Figure 13–Histogram of Residuals of "Best Fit" Straight Line Compared to a Normal Distribution – Serial 100

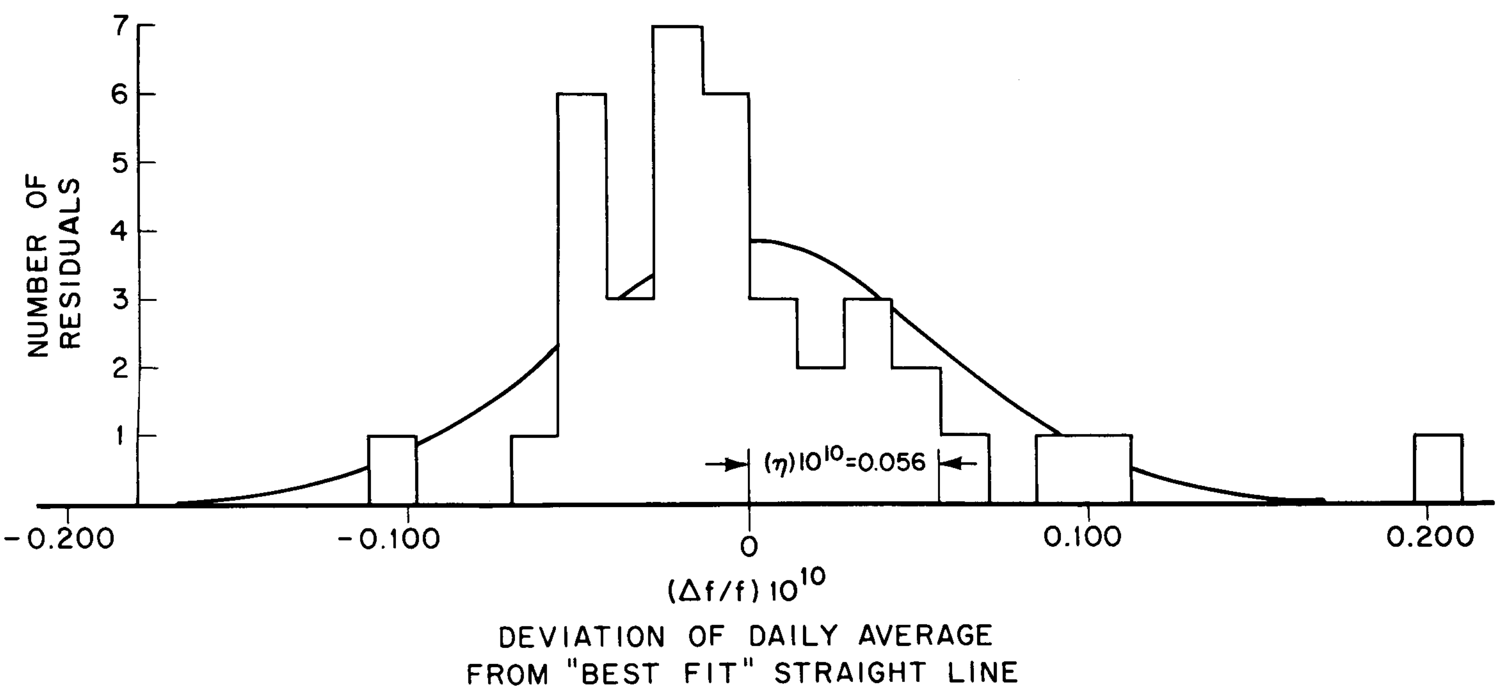


Figure 14–Histogram of Residuals of "Best Fit" Straight Line Compared to a Normal Distribution – Serial 106

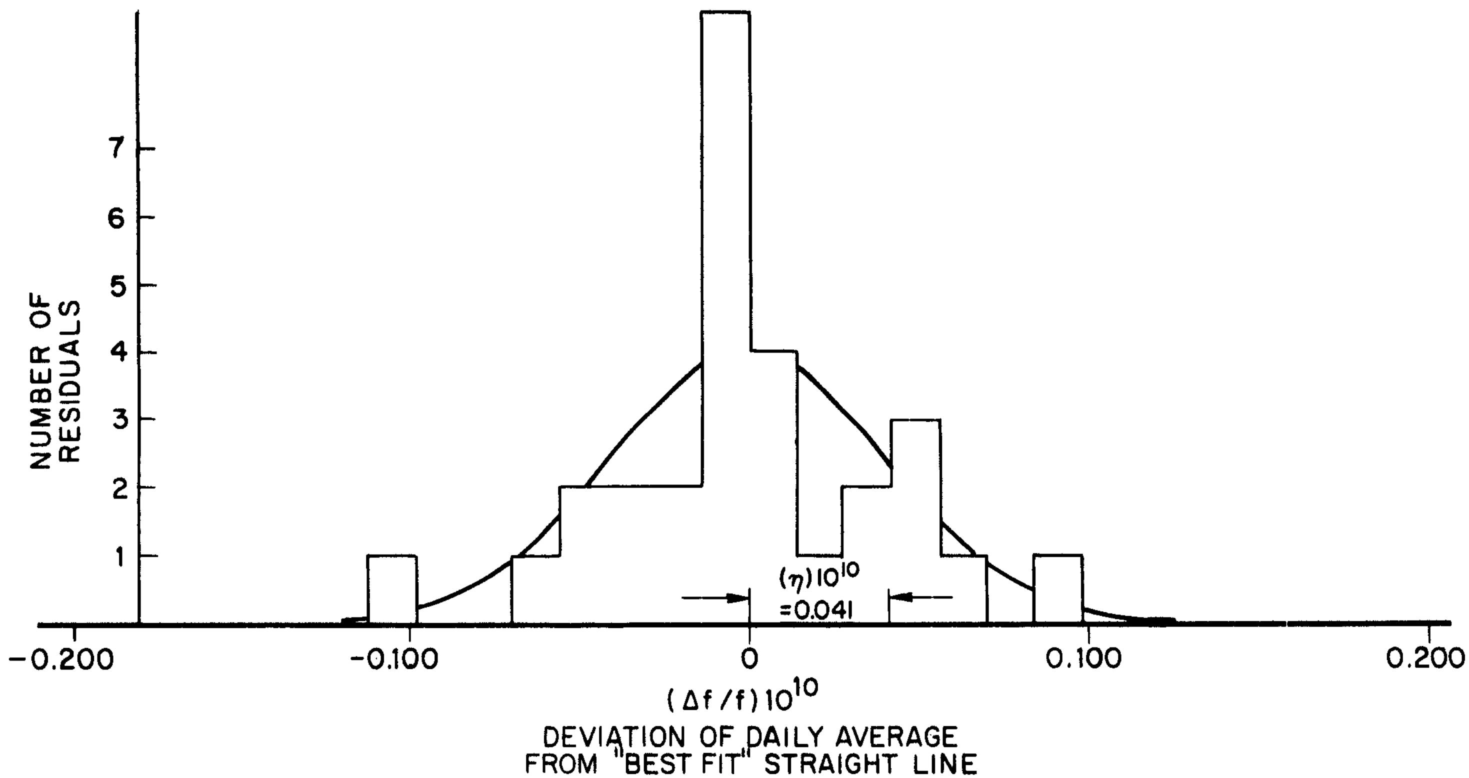


Figure 15–Histogram of Residuals of "Best Fit" Straight Line Compared to a Normal Distribution – Serial 107

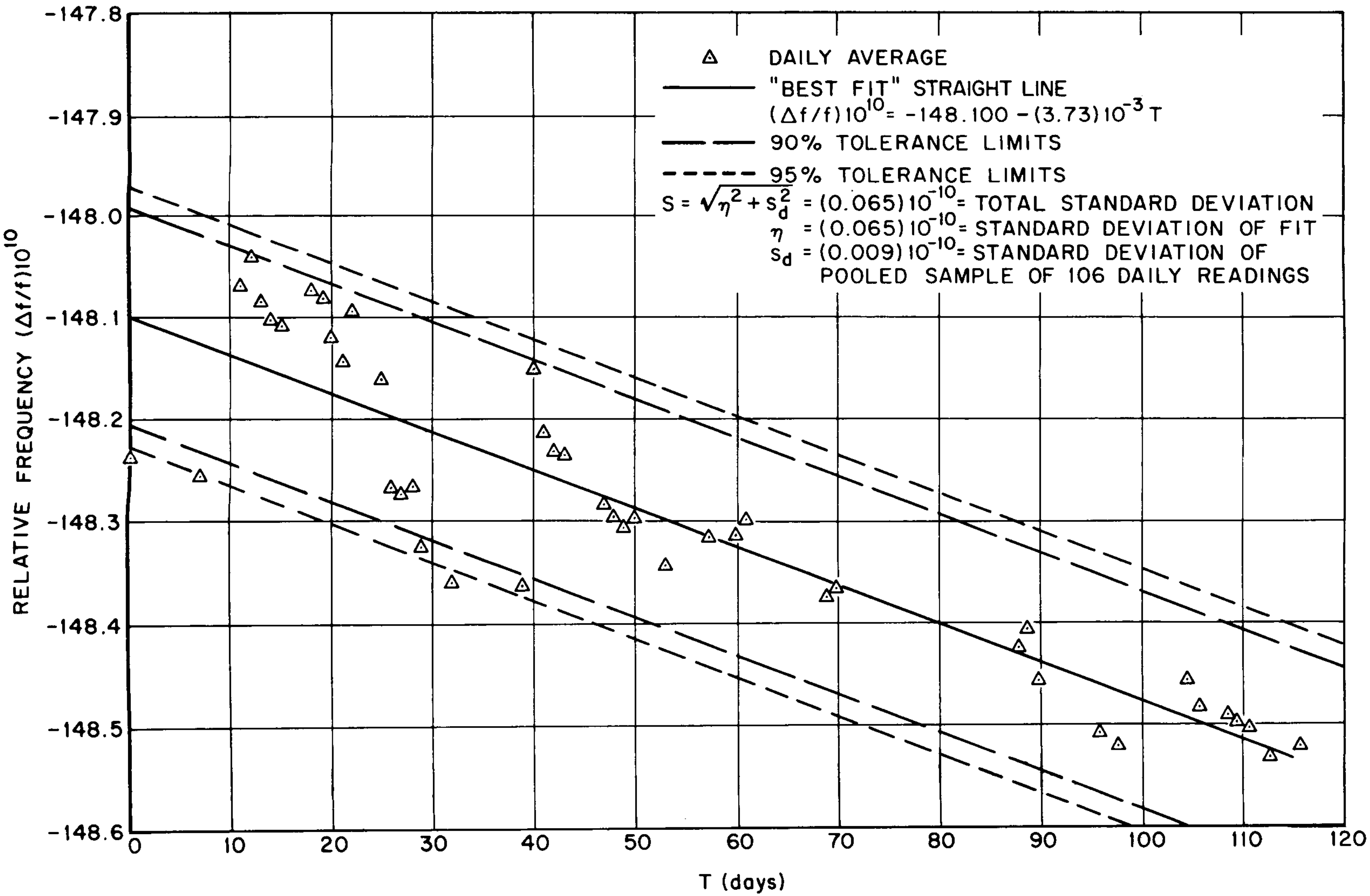


Figure 16–90% and 95% Tolerance Intervals for the Deviations of the Daily Readings from the "Best Fit" Straight Line - Serial 100

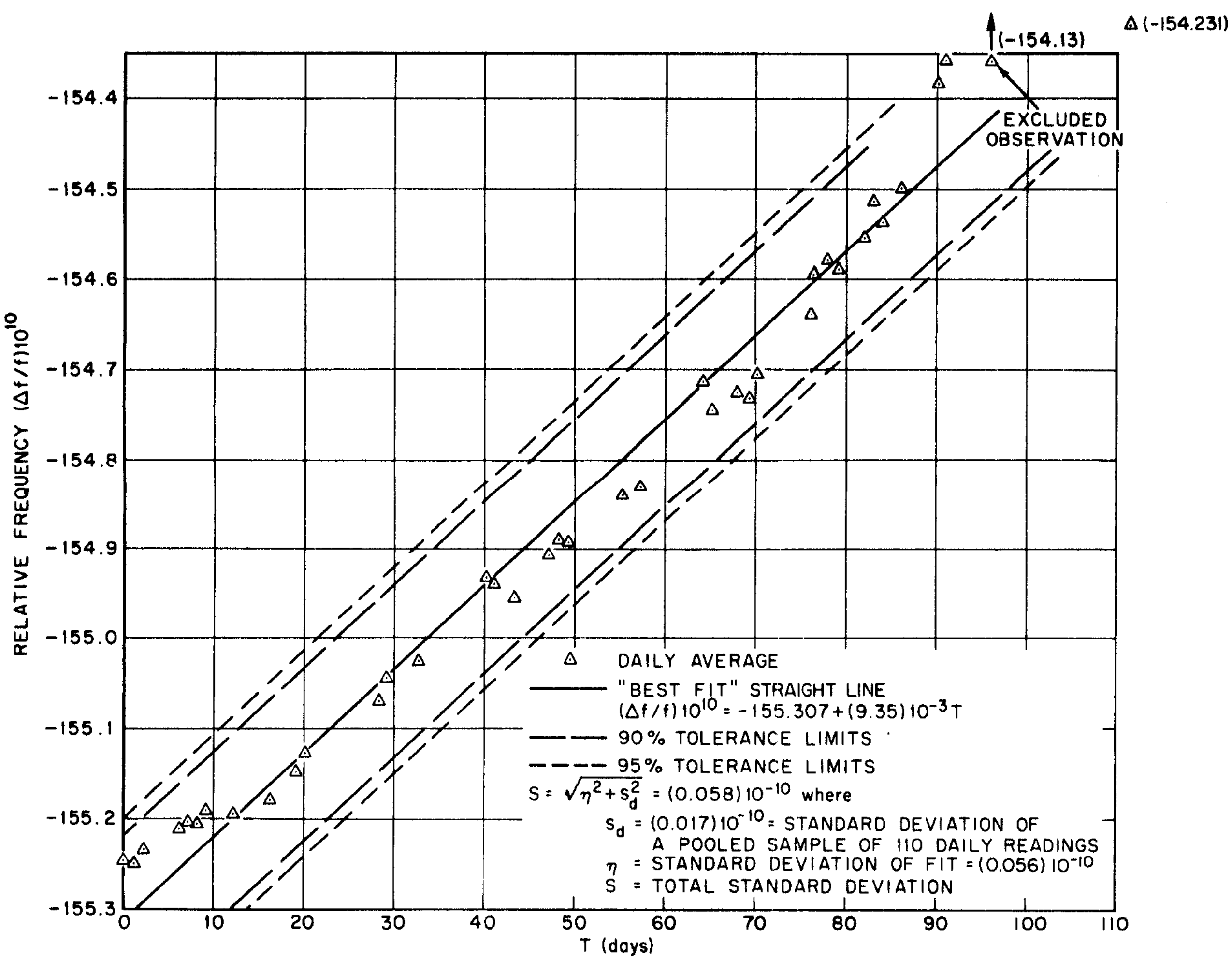


Figure 17–90% and 95% Tolerance Intervals for the Deviations of the Daily Readings from the "Best Fit" Straight Line - Serial 106

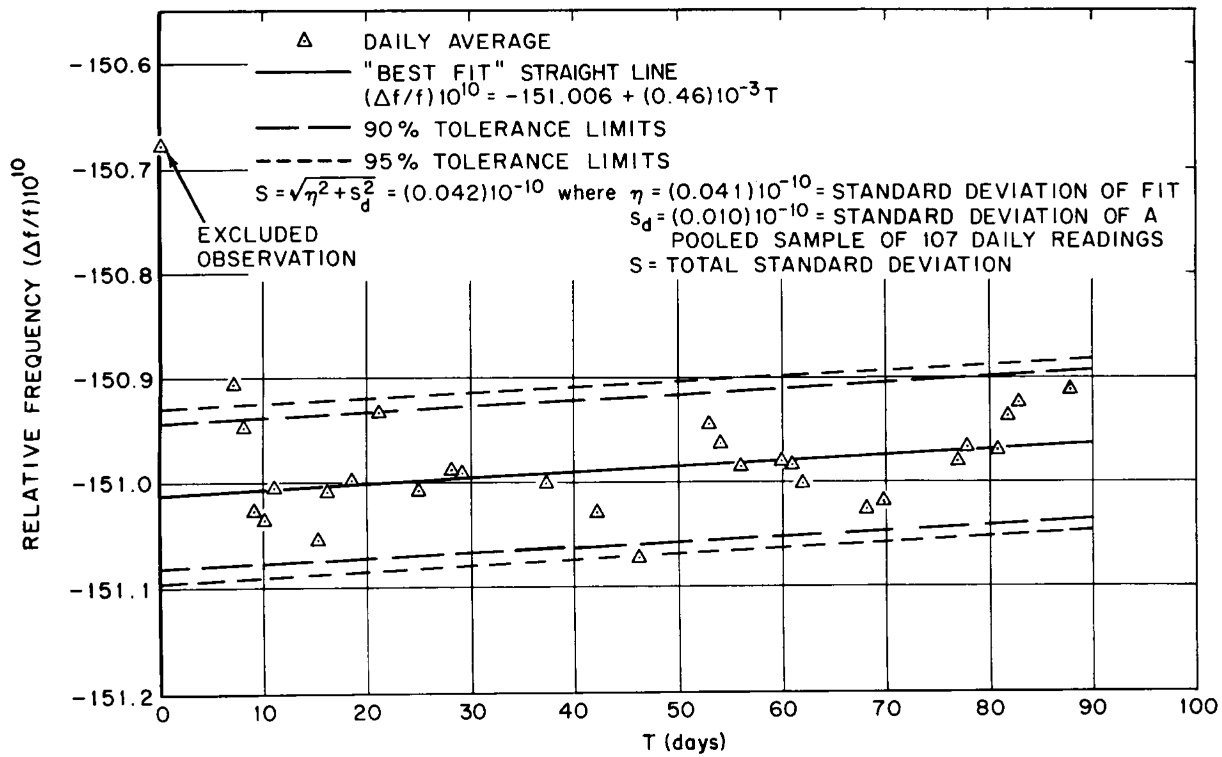


Figure 18–90% and 95% Tolerance Intervals for the Deviations of the Daily Readings from the "Best Fit" Straight Line - Serial 107

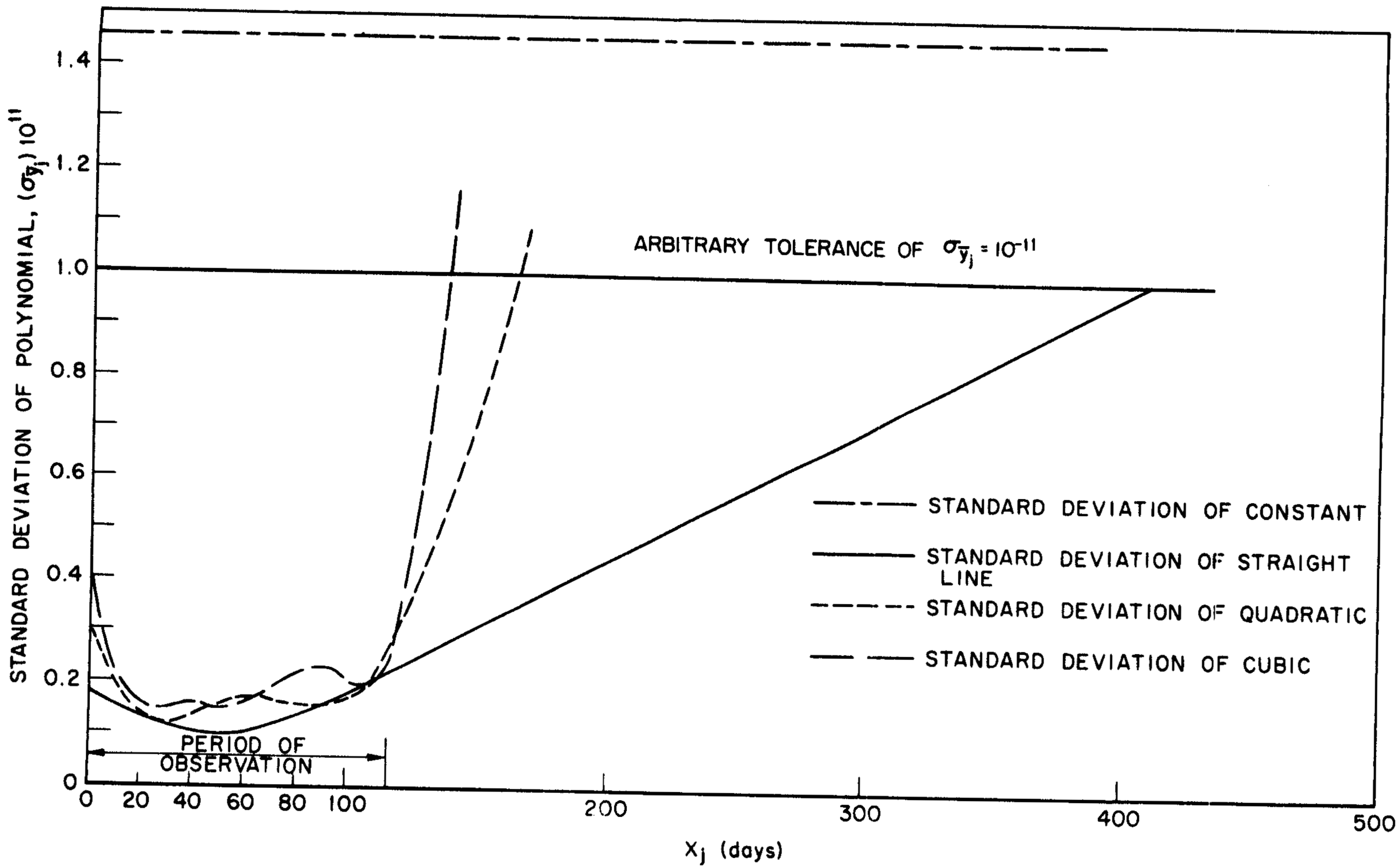


Figure 19–Standard Deviation of the Polynomial VS Time – Serial 100

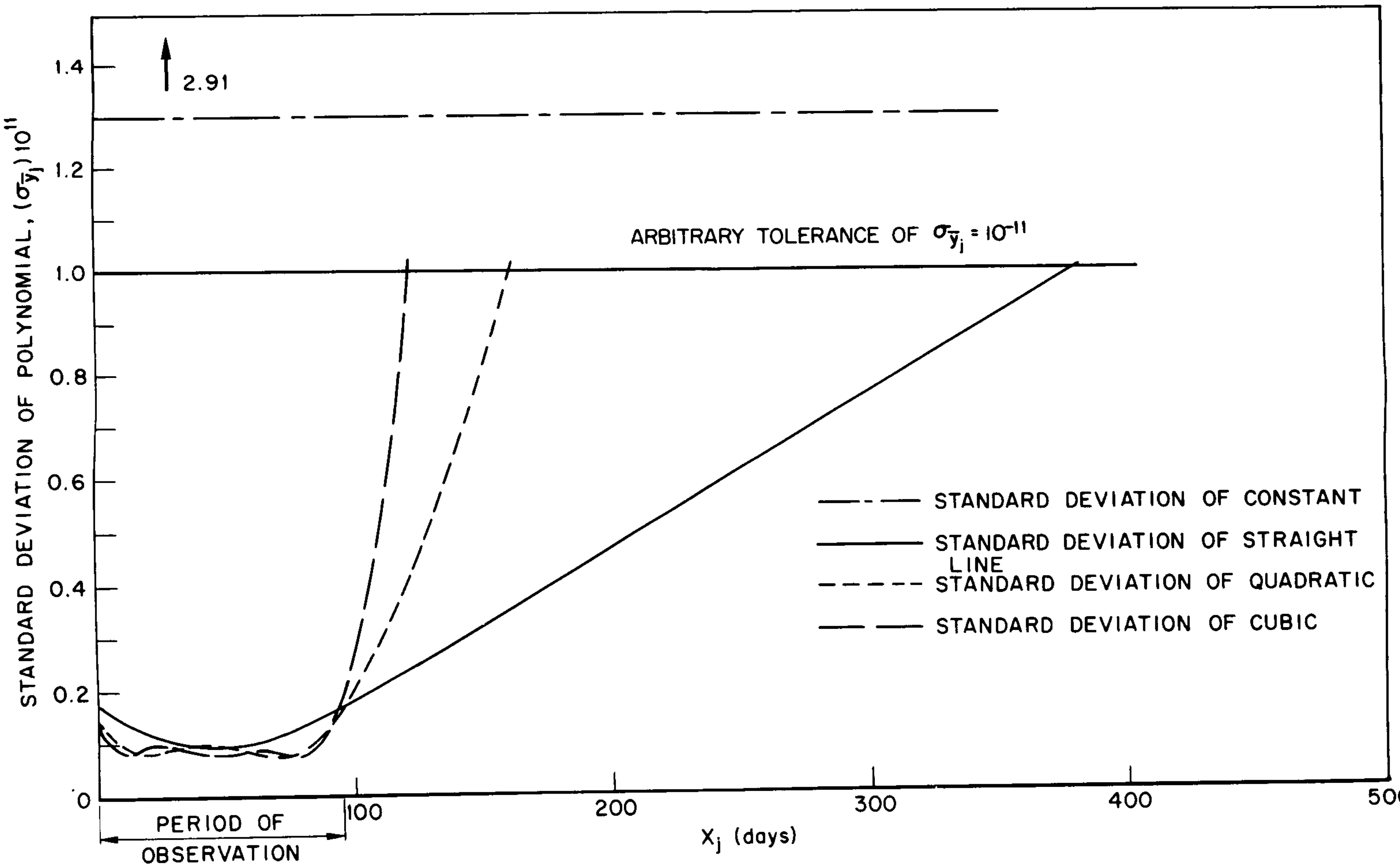


Figure 20–Standard Deviation of the Polynomial VS Time – Serial 106

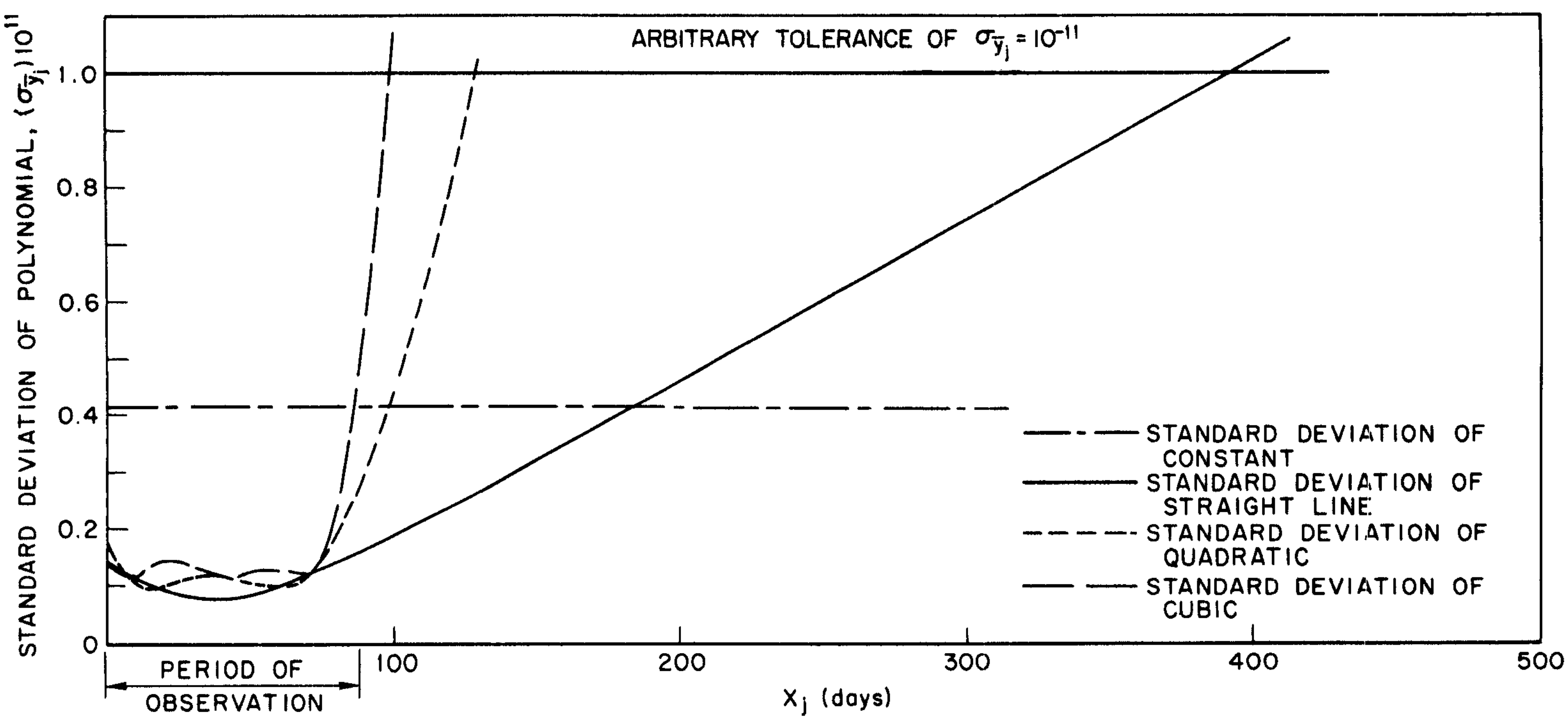


Figure 21—Standard Deviation of the Polynomial VS Time – Serial 107

REFERENCES

1. A. H. Morgan, E. L. Crow, and B. E. Blair, International Comparison of Atomic Frequency Standards Via VLF Radio Signals, Radio Science Journal of Research NBS/USNC-URSI Vol. 69D, No. 7, July 1965.

2. Report of Calibration, Rubidium Frequency Standard, Varian Associates, Model R-20, Serial 100, U. S. Dept. of Commerce, NBS, Institute for Basic Standards, Boulder, Colorado 80301, December 20, 1965.

3. Report of Calibration, Rubidium Frequency Standard, Varian Associates, Model R-20, Serial 106, U. S. Dept. of Commerce, NBS, Institute for Basic Standards, Boulder, Colorado 80301, December, 1965.

4. Report of Calibration, Rubidium Frequency Standard, Varian Associates, Model R-20, Serial 107, U. S. Dept. of Commerce, NBS, Institute for Basic Standards, Boulder, Colorado 80301, December, 1965.

5. F. O. Vonbun and W. D. Kahn, Tracking Systems, Their Mathematical Models and Their Errors, Part 1 - Theory Technical Note, D-1471, NASA, Goddard Space Flight Center, Greenbelt, Maryland.